1. Introduction

The perfect Arrow-Debreu world does not exist. There are several market failures that deviates actual equilibrium from what would arise from its presumptions. Many of the assumptions introduced in this model do not hold in reality and all economies end up converging to an equilibrium that is not optimal. Such assertion is also true for the regional equilibrium in most countries. Of course, the Brazilian economy falls on this more general case and the national regional equilibrium is not optimal under the definition of the Arrow-Debreu world.

Most of the economic assumptions that bear the proof that there is a tendency for equality of per capita income among regions, as the one found in Pessoa (1999) and Barros (2007) fail. Even preferences towards labour and leisure vary from region to region and, in countries like Brazil, from State to State. Nevertheless, these differences are not enough to generate regional disparities that can be found in a country like Brazil. Therefore, some failures on the optimal allocation of resources among regions is necessary to explain such disparities.

Many are the factors justifying these failures in the real world. Whenever these violations of assumptions are introduced, equilibrium conditions of the model change and some conclusions on the regional equilibrium also can be eroded. Nevertheless, not all changes move the economy to an equilibrium that implies in a regional problem or inefficiency. It is possible that the economy leaves an optimal and regionally even equilibrium to another one that also has regional equality, even if it is not economically optimal. Of course, it is also possible that there is a movement from an equilibrium with regional equality to another one with regional inequality, but which does not have a regional problem or inefficiency.

The recent literature on regional economics, which has emphasized the outcome of regional inequality, has been built on changes of the basic assumptions of the Arrow-Debreu model. The models in Fujita, Krugman and Venables (2001) and Krugman (1991) all have such feature. Normally they introduce either the existence of increasing return to scale or transaction costs as

---

1See Barros and Magalhães (2007) and for a presentation of some indicators of the regional disparity in Brazil.
the sources of regional imbalances. Nevertheless, they use very specific model framework and often mix more than one market failure, so that it is not easy to disentangle their individual impacts. Therefore, to build a clearer relationship between each market failure and its role to the nature of regional inequality is still a challenge for economic theory.

It will not be the role of this paper to forward a formal proof of the relationship of each market failure to regional inequality. Nevertheless, the following sections present a discussion of the ability the existence of infrastructure and natural resources have to generate regional inequality and if such inequality implies the existence of a regional problem or inefficiency. The paper is organized as following. Section 2 presents a short discussion of the regional problem, inefficiency and disparity. Section 3 forwards a simple model which shows formally that regional disparity can emerge from the existence of infrastructure and natural resources. Section 4 summarizes the major conclusions.

2. Concepts of regional problem, disparity and inefficiency

In the regional literature, it is common to confuse the concepts of regional disparity or inequality, regional problem and regional inefficiency. Only the two last situations would justify regional policies, as it is possible to improve standards of living through government interventions when they exist. This confusion leads to inadequate policy proposals and misplaced justification for them. Therefore, as the presentation of one source of regional disparity is the object of next sections, it is crucial to understand its consequence for the existence of regional problem and inefficiency.²

A regional disparity exists when:

\[ x^a = \frac{1}{n} \sum_{i=1}^{n} X_i^a \neq \frac{1}{m} \sum_{i=1}^{m} X_i^b = x^b \]  

(1)

Where \( X_i^a \) and \( X_i^b \) are representing the incomes of all \( n \) and \( m \) individuals living in regions \( a \) and \( b \), respectively. Therefore, \( x^a \) and \( x^b \) are per capita income in these two regions. In words, this means that there is a regional disparity when average per capita income differs in the two regions. It should be noticed that under this concept it is possible that \( x^a > x^b \), although the GDP per capita in region \( a \) is smaller than the GDP per capita in region \( b \). If there are only two regions in the economy, this would happen if a reasonable share of the per capita GDP in region \( b \) becomes income for agents on region \( a \). Property of factors of production by such agents working in region \( a \) would justify such

² See Barros (2004) for a formal and detailed discussion of the differences on these concepts.
reversion. Nevertheless, although this possibility is not irrelevant, it will not be the focus here.³

Regional inefficiency, in its turn, exists when there is at least one alternative allocation of factors of production among regions that could make at least one agent better off than he/she is on the current one, without making anyone else worse off, relying only on distribution of additional production or redefinition of total bundle of goods and services produced.

The notion of regional problem is similar to the one of regional inefficiency, but its focus is on the set of individual attributes. Assume that there are two non-negligible sets of individuals, A and B, each one composed of persons living in the same region, but each one settled in one region, also nominated as A and B. Each one of these sets is formed by people who would prefer to have the standard of living of people with all their attributes, but living in the other region. If one of these sets is bigger than the other, then it is possible to say that there is a regional problem. The concept of a regional problem is more complex than the one of efficiency because it allows for differences in per capita income and GDP that are not enough to offset the role of local amenities on welfare.

3. Failure in the free flow of all factors of production

Free flow of factors of production is a basic assumption to generate equality of per capita GDP in the Arrow-Debreu models type. Nevertheless, this assumption is only an approximation, as in reality there is always some transaction costs to move any factor of production among regions. Transport and contractual costs, or even taxes, are some of these costs. Whenever labour is the factor of production to move, transport is the most obvious cost. Nevertheless relocation costs, such as transaction costs for house purchase/rent, car and all durable goods purchases are also important. Capital movement normally leads at least to taxes and contractual costs. Often it also implies in transportation costs, if it is embodied in goods that can be used productively. Natural resources, whenever they can be moved, demand transportation costs. Therefore, in reality all movements of factors of production have some cost. If there is not free flow of factors of production, it is possible to have different equilibrium per capita GDP. Furthermore, the emerging disparity depends on the relative availability of the fixed factors of production.

A simple model can show that inequality can emerge in a static framework, when there is not free flow of at least one factor of production. Suppose that there is a country with two regions, which will be called here as region \( a \) and region \( b \), respectively. Each one of them produces two goods,

³ In the literature on the Brazilian regional disparities, the regional disparities often relies on both of these concepts, taking into account per capita GDP or per capita personal disposable income.

\[ \text{Revista Brasileira de Estudos Regionais e Urbanos} \]
which will be called output 1 and output 2. Firms of both regions face similar production functions for each good, which are defined as:

\[ Y_{ji} = A_{ji}^\alpha K_{ji}^\beta_1 L_{ji}^{1-\alpha-\beta_1} \]  \hspace{1cm} (2)

and:

\[ Y_{zi} = A_{zi}^\alpha K_{zi}^\beta_2 L_{zi}^{1-\alpha-\beta_2} \]  \hspace{1cm} (3)

where \( Y_{ji} \) is the output of good \( j \) in region \( i \), \( K_{ji} \) is the amount of capital used in production of good \( j \) in region \( i \), and \( L_{ji} \) is the amount of labour used in production of good \( j \) in region \( i \). In this economy there is a third factor of production, which cannot move among regions. It can be the available natural resources or infrastructure,\(^4\) for example. It is represented by \( A_{ji} \) in equations (2) and (3). Both these potential factors represented by \( A_{ji} \) could have this same logical representation, as they do not move from one region to the other. All the variables are non-negative to have economic meaning, so that by assumption, \( Y_{ji} \geq 0 \), \( A_{ji} \geq 0 \), \( K_{ji} \geq 0 \), and \( L_{ji} \geq 0 \).

The parameters \( \alpha \) and \( \beta_j \) are the output share for good \( j \), with respect to \( A_j \) and \( K_j \), respectively. They satisfy the restrictions \( 0<\alpha<1 \), \( 0<\beta_1<1 \), \( 0<\beta_2<1 \), \( \alpha+\beta_1<1 \) and \( \alpha+\beta_2<1 \), so that all factors of production has a positive contribution for total output. As technology flows freely among regions (assumption of free flow of information), the parameters \( \alpha \) and \( \beta_j \) are the same for the two regions. Nevertheless, the availability of natural resources (or economic infrastructure) may differ in the two regions, as they cannot flow from one region to the other. Therefore, it is expected that \( A_{ja} \neq A_{jb} \), for both \( j =1 \) or \( j=2 \). Nonetheless, a simplifying assumption is introduced here and the following equality is supposed to hold:

\[ A_{2a} = A_{1b} = A_{2b} = 1 \]  \hspace{1cm} (A1)

Also, as an assumption to show the role of differences in availability of factors of production, as they are not free to flow among regions, it is assumed that \( A_{1a}>1 \).

Another assumption introduced is that \( \beta_1>\beta_2 \). This implies that the good which has a higher availability of the natural resources used in its production within one of the regions is more capital intensive in its technology of production. Particularly, this good is output 1 in region \( a \). Such assumption can show the effect of a higher elasticity of output with respect to capital in the

\(^4\) If \( A_{ji} \) represents the stock of infrastructure, it is reasonable to assume that non-economic factors determined its spatial distribution. Politics normally is a key determinant.
region which has a higher availability of the non-moving factor of production $A_1$.

Another important simplifying assumption is that both goods are tradeables and they have their prices settled in the world market, such that $P_{1a}=P_{1b}=P_{2a}=P_{2b}=1$. Of course, the particular assumption that prices are equal to one is not restrictive, as a re-definition of units could lead to such equality.

In equilibrium, we have that:

$$L_{1a} + L_{2a} + L_{1b} + L_{2b} = L_T \quad (4)$$

and:

$$K_{1a} + K_{2a} + K_{1b} + K_{2b} = K_T \quad (5)$$

where $K_T$ and $L_T$ are both fixed and positive.

Given the production functions defined in equation (2), it is possible to determine the following arbitrage conditions in the labour market, which arise from first order condition of profit maximization by firms:

$$\left(1 - \alpha - \beta \right) \frac{Y_{1a}}{L_{1a}} = w \quad (6a)$$

$$\left(1 - \alpha - \beta \right) \frac{Y_{1b}}{L_{1b}} = w \quad (6b)$$

$$\left(1 - \alpha - \beta \right) \frac{Y_{2a}}{L_{2a}} = w \quad (6c)$$

$$\left(1 - \alpha - \beta \right) \frac{Y_{2b}}{L_{2b}} = w \quad (6d)$$

where $w$ is the real wage rate and the other variables and parameters are as previously defined.

The market for the fixed factor of production also is susceptible to arbitrage. As the production functions are all homogeneous of first degree, Euler Equation assures that they all generate zero profit when there is payment of factors of production by their marginal product. Therefore, no entrepreneur could pay more than its marginal product for the fixed factor, otherwise he/she will not be able to pay the marginal product for the other factors of production. In the same way, under perfect competition, they will not be able to pay the fixed factor less than its marginal product, otherwise other entrepreneurs will profit from a marginal bidding for this factor. Therefore, the equilibrium price for the fixed factor is also its marginal product. Furthermore, if there is no cost
to move labour and capital between regions, if the price of the fixed factor of production is higher in one region than in the other, entrepreneurs will move its production from the region with higher cost for this factor to the region with lower cost. Therefore, only the same cost for these factors in the two regions will be an equilibrium. These conclusions imply that:

\[ \alpha \frac{Y_{1a}}{A_{1a}} = \alpha \frac{Y_{1b}}{A_{1b}} = \rho_1 \]  
\[ (7a) \]

\[ \alpha \frac{Y_{2a}}{A_{2a}} = \alpha \frac{Y_{2b}}{A_{2b}} = \rho_2 \]  
\[ (7b) \]

where \( \rho_1 \) and \( \rho_2 \) are the returns to the fixed factors of production 1 and 2, respectively. Arbitrage assures that these returns are exactly the same in the two regions, for each of the fixed factors of production. Nevertheless, there is no reason to suppose that \( \rho_1 = \rho_2 \), so that the most general case is that they actually differ.

Assumption (A1) and equation (7) implies that:

\[ Y_{1a} = A_{1a} Y_{1b} \]  
\[ (8) \]

and:

\[ Y_{2a} = Y_{2b} \]  
\[ (9) \]

Consequently, from equations (8), (9) and (6):

\[ L_{2a} = L_{2b} \]  
\[ (10) \]

and:

\[ L_{1a} = A_{1a} L_{1b} \]  
\[ (11) \]

Let us assume that there is only one firm producing each of the outputs in each region and define regional disparity as \( \sigma \), such that:

\[ \sigma = \frac{Y_{1a} + Y_{2a}}{L_{1a} + L_{2a}} - \frac{Y_{1b} + Y_{2b}}{L_{1b} + L_{2b}} \]  
\[ (12) \]

This concept is as straight as possible. Regional disparity is defined mathematically as the difference of per capita output in the two regions. Of course this becomes disparity in per capita income only if there is no cross
property of factor of production between the two regions. All the property of factors of production employed in one region belongs to agents in that same region. The literature normally relies on per capita GDP to unveil disparity, so that the idea of per capita output is a good theoretical concept to build the representation of regional disparity, as done in equation (12).

Using equations (8) to (11) to substitute for $Y_{1b}$, $Y_{2b}$, $L_{1b}$, $L_{2b}$, it is possible to get:

$$\sigma = \frac{Y_{ia} + Y_{2a} - Y_{1a} + A_{1a}Y_{2a}}{L_{ia} + L_{2a} - L_{1a} + A_{1a}L_{2a}}$$  \hspace{1cm} (12')$$

Rearranging terms, this equation can be re-written as:

$$\sigma = \frac{(A_{ia}-1)(Y_{ia}L_{2a} - Y_{2a}L_{ia})}{(L_{ia} + L_{2a})(L_{ia} + A_{ia}L_{2a})}$$  \hspace{1cm} (12'')$$

Substituting $Y_{1a}$ and $Y_{2a}$ from equations (6a) and (6c) in this equation:

$$\sigma = \frac{(A_{ia}-1)\mu L_{1a}L_{2a}\left(\frac{1}{1-\alpha-\beta_1} - \frac{1}{1-\alpha-\beta_2}\right)}{(L_{ia} + L_{2a})(L_{ia} + A_{ia}L_{2a})}$$  \hspace{1cm} (13)$$

As $A_{1a}>1$, as determined by assumption, the fact that all variables only have an economic meaning when they are positive implies that $\sigma>0$ if $\beta_1>\beta_2$.\footnote{In fact variables have economic meaning when they are non-negative. Nevertheless, Inada conditions for optimization of firms in models with Cobb-Douglas production functions assure that the equilibrium values of all factors of production, in each firm and in each region, are positive.}

Furthermore, there is no regional disparity only if $\beta_1=\beta_2$ and the production functions of both goods are equally intensive in capital. It is also possible to see that if $\beta_1<\beta_2$, the region with higher availability of one of the fixed factor of production has lower per capita output. These conclusions support the following statement:

**Proposition 1:** An economy with two regions that produces two different goods can generate regional disparity if one of these goods is more intensive in capital and there is different availability of a factor of production that cannot move between the two regions.

This proposition can be generalized to more than two regions and more than three factors of production used by each firm, with more than one fixed. It would, then state:

**Proposition 2:** An economy with many regions that produces many different goods can have regional disparity if there are different availability of non-moving factors of production that generate regional specializations such
that the bundles of goods produced in the many regions have different capital intensities among them.

4. Conclusions

Suppose that there is a public sector that can move some factors of production from one region to the other, through taxes and subsidies, for example, the new allocation of resources will not be socially optimal. Decreasing marginal return for each factor of production assures that the region which gets more moving factors of production will add up to its total output, given the prevailing world prices, less than the region that loses factors of production will face in reduction of its output, as a consequence of this fall in the availability of factors of production. Therefore, there is no way to gain economic efficiency through regional policies. This implies that there is not regional inefficiency in this economy, though there is regional disparity.

It is worth noting that the emerging regional disparity also does not lead to the existence of a regional problem. By arbitrage, all individuals that move from one region to the other will be able to get the same income as before migration. As the payments for capital and labour are exactly the same in the two regions, by the assumption of perfect arbitrage among the regions, these two factors of production belonging to any individual will have the same income, if he/she is employing them in any of the regions in the country. Therefore, if amenities are exactly the same in the two regions, as assumed before, there is no reason someone would prefer the standard of living of any person with the same attributes that lives in the other regions.

It was seen that by arbitrage, the returns to any of the fixed factors of production are also exactly the same in all regions, even when their absolute availabilities differ. Therefore, any individual that possesses a certain amount of one of these factors of production in one region can sell this amount and buy exactly the same amount in other region and have exactly the same income as they did before. The hypothesis of no transaction cost assures that this trade will assure the same amount of the fixed factor of production in the two regions. The fact that the rates of returns are exactly the same also assures that the prices of these fixed factors of production in the two regions are equal. Therefore, the introduction of the fixed factor of production does not alter the conclusion about the inexistence of a regional problem, when there are only moving factors of production.

These conclusions imply that this violation of Arrow-Debreu basic assumptions, which is the possibility of non-free movement of at least one factor of production, already justify the appearance of regional disparity. Nevertheless, this will not lead to the emergence of a regional problem or inefficiency. Therefore, the simple existence of regional disparity in this case does not justify
the employment of regional policies, as they will reduce social welfare if its source is a fixed factor of production.

References


